Crisis in the geometry development and its social consequences

Yuri A Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences,
101-1, Vernadskii Ave., Moscow, 119526, Russia.
e-mail: rylov@ipmnet.ru
Web site: http://rsfq1.physics.sunysb.edu/~rylov/yrylov.htm
or mirror Web site: http://gasdyn-ipm.ipmnet.ru/~rylov/yrylov.htm

Abstract

The reasons of the crisis in the contemporary (Riemannian) geometry are discussed. The conventional method of the generalized geometries construction, based on a use of the topology, leads to a overdetermination of the Riemannian geometry. In other words, at the Riemannian geometry construction one uses the needless information (topology), which disagrees with other original axioms. The crisis manifests in the fact, that the mathematical community cannot see and does not want to see the overdetermination of the Riemannian geometry. The most geometers-topologists deny the alternative method of the generalized geometry construction, which does not uses the topology, because it does not contain theorems. Most geometers see the geometry presentation as a set of definitions and theorems. They cannot imagine the geometry presentation without customary theorems. As a result the most clever topologists, which have acknowledged the negligible role of the topology in the geometry construction and inconsistency of the conventional method of the generalized geometry construction, appear in the difficult situation (conflict with the mathematical community).

1 Introduction

I wrote on crisis in geometry [1]. The fact is that the contemporary (Riemannian) geometry is overdetermined and hence it is inconsistent. Although the character of inconsistency is known long ago, the mathematical community as a whole does not want to acknowledge these inconsistencies and bypasses them. The content of the crisis lies in the fact that the mistakes in geometry (more exact in the construction of generalized geometries) are not acknowledged and not corrected, but not in the existence of mistakes in themselves. As a result the geometry is developed in the direction, which leads in the blind alley. Why does such a situation take place and which are consequences of the crisis? The presented paper is devoted to this question.

Let us note that it the second crisis in the geometry. The first crisis took place in the second half of the 19th century, when the mathematical community did not want to acknowledge the reality of the non-Euclidean geometries. This crisis had exhausted itself somehow, when the non-Euclidean (Riemannian) geometry begun to be applied in the general relativity. But the analysis of the crisis reasons has not been produced (at any rate, I know nothing about such an analysis).

At first, I connected an appearance of the crisis with the strongly crusted preconception, that the straight is a one-dimensional line in any generalized geometry. It followed from this statement that the one-dimensional line is the most important object of any generalized geometry. In its turn this fact leads to the conclusion that the topology, founded on the concept of one-dimensional curve, lies in foundation of a geometry, and it should be used in construction of a generalized geometry. All conventional methods of the geometry construction are founded on the essential employment of the topology. It means that at first one constructs a topological space, and the geometry was constructed on the basis of the topological space. Overcoming the preconception on the one-dimensionality of the straight, one can construct a more general and simpler method of the generalized geometries construction. This alternative method is founded on the deformation principle [2]. This method admits, that the straight may be a surface (tube), and it does not need the topology [3] for construction of a generalized geometry. The substance of the alternative method may be expressed by the words: "Any generalized geometry may be obtained as a deformation of the proper Euclidean geometry. The alternative method, applied for the Riemannian geometry construction, is free of the well known defects of the conventional method, i.e. it does not contain such properties of the Riemannian geometry as absence of the absolute parallelism and the convexity problem. All scientists understood that these properties of the Riemannian geometry were its defects, but they could not overcome them without a use of the alternative method.

The fact, that the alternative method gives another results than the conventional one, means that at least one of methods is erroneous. To use the conventional method, one needs many different constraints, restricting a possibility of its application. One needs the dimensionality of the space, its continuity and a continuous coordinate system in it. For application of the alternative method one does not

need neither dimensionality, nor the space continuity. The coordinate system is not needed also. One needs only the distance function $\rho(P,Q)$, or the world function $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, which satisfies the only condition

$$\sigma(P,P) = 0 \tag{1.1}$$

Even the symmetry condition

$$\sigma(P,Q) = \sigma(Q,P) \tag{1.2}$$

is not an obligatory condition [4]. The alternative method does not contain any logical reasonings, whereas at application of the conventional method of the generalized geometry construction one needs to provide consistency of all used suppositions. It needs the significant efforts. Consistency of all constraints may be provided not always. For the Riemannian geometry the consistency is violated because of the supposition, that the straight is an one-dimensional curve (or because of the application of the topology at the geometry construction, what is essentially the same). The fact is that at construction by means of the right (alternative) method the geodesic, passing through the point P, in parallel to the vector at the point P, is one-dimensional. However, in the case, when the straight (geodesic) is passed through the point P, in parallel to the vector at the other point P_1, it is not one-dimensional, in general [5]. To avoid the non-one-dimensionality of the straight, the absolute parallelism is forbidden in the Riemannian geometry. Thus, it is evident that the alternative method is true, whereas the conventional one is questionable. In the case, when they lead to different results, one should to prefer the alternative method.

I believed that the construction of generalized geometries by means of the alternative method, founded on the deformation principle, admits one to perceive the inadmissibility of the topology application at the generalized geometry construction. But I appeared to be not right in the relation, that I considered the one-dimensionality as an only preconception preventing from the correct construction of the generalized geometry. It appears, that there is once more preconception. I did not guess about this preconception, because I did not meet it at the construction of the tubular geometry (T-geometry), i.e. the geometry constructed on the basis of the deformation principle. I met the preconception on the one-dimensionality of straight at construction of the T-geometry and expended thirty years to overcome it [1]. One can obtain a representation on these preconceptions from consideration of following two syllogisms:

1. According to Euclid the straight is one-dimensional in the Euclidean geometry, hence, the straight is one-dimensional in any generalized geometry.

2. Euclid constructed his geometry, formulating and proving theorems, hence, any generalized geometry should be constructed, formulating and proving theorems.

From the viewpoint of logic the considered syllogisms are not valid, especially if one takes into account the opinion of the ancient Egyptians, supposed that all revers flow northward. Their viewpoint may be considered as a corollary of the third syllogism:

3. The great river Nile flows northward, hence all revers flow northward.

The structure of all three syllogisms is the same. They transform a partial case into the general one without a sufficient foundations. Such a generalization is connected with a narrowness of our experience. There is only one (Euclidean) geometry, and the straight in this geometry is one-dimensional. There is the only geometry, and it constructed by means of formulation and proof of theorems. Finally, the ancient Egyptians knew only one river Nile.

The logical inconsistency of syllogisms did not prevent mathematicians from application of them at construction of any generalized geometry, because in the given case the mathematicians used in their practical activity the associations, but not a logic. In all considered cases we deal with the only object. In this case it is very difficult to distinguish, which properties belong to the object in itself and which properties belong to the method of the object description. When the method of description is considered to be a property of the object in itself, we obtain a preconception. The first syllogism is a reason of the preconception on one-dimensionality of the straight. (The method of the straight description, used by Euclid, was acknowledged as the property of the straight in itself). I knew about this preconception, because I was forced to overcome it. The second syllogism became to be a foundation of the preconception, that all activity of a geometers consists in a formulation and a proof of theorems. (Euclid described the geometry by means of theorems, hence description in terms of theorems is a property of the geometry. If there are no theorems, then there is no geometry.) I had met the second preconception at the following circumstances.

I have submitted my report [3] to the seminar of one of geometric-topological chairs of the Moscow Lomonosov University. It took place in 2001, and my paper [3] was not yet published. I came to the secretary of the seminar, told him about my wish to read a report and gave him the text of the paper. Skimming my paper the secretary of the seminar said thoughtfully: "How strange geometry! No theorems! Only definitions! I believe that it will be not interesting for us." One of leading geometers, which was forced duty bound to read my paper (the paper was, submitted to the International conference on geometry, which took place in Saint-Petersburg in the summer of 2002) said me in private conversation, that he had understood nothing in my paper, because it contained neither axioms, nor theorems. In that time I did not understand his objections. It was quite unclear for me, how one cannot understand such a simple conception as the T-geometry, which did not contain any logical reasonings. Only essentially later I had understand, that here we deal with a preconception. Mathematicians distinguish from other peoples in the relation, that they perceive the geometry exclusively via its formalism. Further I shall try to show formally, how theorems are transformed into definitions, and the demand of the geometry representation in the form of a set of theorems is not more, than a preconception.

Speaking about social consequences of the crisis, I keep in mind as follows. Now the topology is considered to be the most perspective direction of the geometry development. All best geometers are concentrated in the development of this direc-

tion. Having overcame the preconception on the one-dimensionality of the straight and discovering the alternative method of the geometry construction, the topology appears to be a cul-de-sac direction in the geometry development. Many papers on verification of the geometry on the basis of topology appear to be depreciated. However, these papers are considered to be important and perspective as long as the mathematicians do not want to take into account the alternative method and do not acknowledge inconsistencies in the Riemannian geometry.

Let us imagine the following situation. A young talented topologist N has solved a very difficult topological problem. A prestige international prize is declared for solution of this problem. Solving the problem and publishing the results, the topologist N discovers suddenly, that the problem is set incorrectly, because the topology is founded on concepts of the Riemannian geometry, but the Riemannian geometry is overdetermined (i.e. it is inconsistent). The topologist N is awarded this prize quite deservedly, because one did not find mistakes in his solution. But, if the problem is stated incorrectly, it may have no correct solution at all, because different methods of solution may lead to different results. The mathematical community considered that the prize has been awarded correctly, but the topologist N disagree with the opinion of the mathematical community, as far as he understand, that his work may be considered as outstanding one, as long as the mathematical community did not discover the weakness of the ground of the solved problem. What does the topologists N do? If he is a conformist, he may affect ignorance, that he does not discover any incorrectness in the statement of the problem and get quietly the awarded deserved prize. (Whilom the inconsistency of the Riemannian geometry will be discovered!). It is impossible to prove, that he has known about inconsistency of the Riemannian geometry. However, the following problem remain before the topologist N. Further some of his previous papers will be depreciated. How and what for to work in the region of the topological verification of the geometry, if this direction leads to the blind alley?

However, if the topologist N is a scrupulous scientists, but not a conformist, he has only one possibility: to keep away from the opinion of the mathematical community and to abandon from the awarded prize. Should he declare the reason of his decision? It is not a simple question. Declaring the reason of his decision, the topologist N conflicts with the mathematical community, which does not acknowledge the Riemannian inconsistency at this moment. In his time the great Gauss did not risk to conflict with the mathematical community on the question of the non-Euclidean geometry existence. (According to information of the Felix Klein [6] one discovered unpublished manuscripts of Gauss on the non-Euclidean geometry among his papers). However, Gauss worked in many different branches mathematics, he may admit himself to neglect his works in the non-Euclidean geometry. The contemporary topologists works, as a rule, only in topology. Acknowledgment of inconsistencies in the Riemannian geometry is a drama for a topologist. At the same time such an acknowledgment is a very fearless act.

I suspect, that even if the topologists N discussed inconsistency of the Riemannian geometry with his colleagues, he may meet an incomprehension. I have the

following reason for such a suspicion. In the summer 2002 the international conference, devoted to the anniversary of the known Russian geometer A.D. Alexandrov, took place in Saint-Petersburg. Submitting the corresponding report [4], I arrived at the conference. However, the main goal of my participation in the work of the conference was not my report, which appeared to be interesting for nobody. I wanted to discuss with leading geometers of Russia a possibility of the alternative method of the Riemannian geometry construction, which concerns the geometry foundation. It was important for me, because I am not a mathematician, but a physicist - theorist. I succeeded to discuss this question with some leading geometers, but nobody did not understood me (More exactly, only one person had understood me, but he was not a geometer. He was simply a collaborator of the Saint-Petersburg branch of Mathematical institute, where the conference took place). The leading geometers told me very polite, that the question of the geometry foundation had been solved many years ago by Hilbert and other great geometers. In the present time the problem of the geometry foundation is interesting for nobody. In this discussion only the question on the alternative method of the geometry construction was considered. The question of the Riemannian geometry inconsistency was not discussed.

In other time I wanted to discuss the paper [3] at the seminar of the well known mathematician, who had usually a preliminary talk with the potential speaker. During this talk I mentioned that the result of application of the alternative method of the Riemannian geometry construction disagree at some points with the results of application of the conventional method. According to my representation, this fact must intrigue the leader of the seminar in discussion of my report. However, I was told immediately, that the Riemannian geometry cannot be inconsistent. Our talk was finished after this declaration.

Summarizing, I should note that the more talented the topologist, the earlier he faces the inconsistency of the Riemannian geometry, lying in the foundation of the topology and makes the necessary conclusions. A failure in his scientific career may be a result of this inconsistency. Let me note, that the Riemannian geometry inconsistencies may be manifested only on the sufficiently high level of the geometry (topology) development, when new contradictions (other than the convexity problem and the fernparallelism problem) appear. This does not concern the standard formalism of the Riemannian geometry (metric tensor, covariant derivatives, curvature, etc.).

At first site the drama in the scientific career of the topologist N seem to be unreal. However, it has taken place already, puzzling the mathematical community, because nobody cannot imagine that the reason lies not in the character of the topologist N, but in the crisis, which the talented topologist has acknowledged earlier, than other scientists. His behavior in the given situation was very dignified, although it was not clear for his colleagues. I shall not denominate the name of this topologist, although persons close to mathematics and topology may easily determine it.

Farther the pure mathematical questions will be considered, which explain my statement on the overdetermination of the Riemannian geometry. The second sec-

tion is devoted to comparison of the conventional method of the generalized geometry construction with the alternative one. In the third section the question on the Riemannian geometry inconsistency is considered. In the fourth section one considers why the mathematicians discard the T-geometry, constructed on the basis of the deformation principle.

2 Comparison of the conventional and alternative methods of the geometry construction

The conventional method of the Riemannian geometry is as follows. One considers m-dimensional surfaces in the n-dimensional Euclidean space ($m < n$). Those properties of m-dimensional surfaces, which do not depend on the dimension n of the accommodating Euclidean space, declared to be an internal geometry of the m-dimensional surface. It is the Riemannian geometry of the m-dimensional space. The Riemannian geometries are restricted by the constraint, that the Riemannian space can be embedded isometrically into the Euclidean space of sufficiently large dimension. The geometry which cannot be embedded isometrically in the Euclidean space, cannot be constructed by this method. Besides the Riemannian geometries are continuous, that is connected with the application of the continuous coordinate system at the construction of the Riemannian geometry. A use of the conventional method of the generalized geometry construction, i.e. the geometry more general, than the Riemannian one, contains a series of resricitions on the generalized geometry. In particular, such a constraint is the embeddability of the space with the generalized geometry into the Euclidean space of the sufficiently large dimension. Besides, such a generalized geometry contains such a characteristic of the geometry as the dimension, which is a natural number n. Necessity of some dimension n at the generalized geometry seems to be something as a matter-of-course, although, in reality the dimension is a corollary of the applied method of the generalized geometry construction, when one uses the concept of a manifold, which is a coordinate system, consisting n independent coordinates. The fact, that the dimension is not a necessary property of the generalized geometry (it is rather the means of description), follows from the fact that there is an alternative method of the generalized geometry description, where the concept of dimension may be not introduced.

The alternative method of the generalized geometry description is founded on the deformation principle, which states that any generalized geometry can be obtained as a result of a deformation of the proper Euclidean geometry. Any deformation means a change of distance between the points of the space. Any deformation of the proper Euclidean space generates some generalized geometry. It is produced as follows. One proves the theorem, that the proper Euclidean geometry may be formulated in terms and only in terms of the world function [3]. (The world function is a half of the squared distance between the two points of the space). It follows from the theorem, that any geometrical object $\mathcal{O}_{\mathrm{E}}$ and any statement $\mathcal{R}_{\mathrm{E}}$ of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be expressed in terms of the world function σ_{E}

of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ in the form $\mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ and $\mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ respectively. *The set of all geometrical objects $\mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ and relations $\mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ between them forms the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$.* To obtain the corresponding relations of the generalized geometry $\mathcal{G}$ it is sufficient to replace the world function σ_{E} of the proper Euclidean geometry with the world function σ of the generalized geometry $\mathcal{G}$:

$$\mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}}) \rightarrow \mathcal{O}_{\mathrm{E}}(\sigma), \qquad \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}}) \rightarrow \mathcal{R}_{\mathrm{E}}(\sigma)$$

Then the set of all geometrical objects $\mathcal{O}_{\mathrm{E}}(\sigma)$ and relations $\mathcal{R}_{\mathrm{E}}(\sigma)$ between them forms the generalized geometry $\mathcal{G}$.

The alternative method of the geometry construction supposes, that the generalized geometry $\mathcal{G}$ is determined completely by its world function, and any statement of the generalized geometry $\mathcal{G}$ can be obtained from the corresponding statement of the Euclidean geometry. In this case for construction of the generalized geometry $\mathcal{G}$ one does not need a coordinate system. One does not need either its dimension, or any another information on topology of the generalized geometry $\mathcal{G}$. The dimension and topology (if they exist) can be obtained from the world function σ of the generalized geometry $\mathcal{G}$. The method of determination of the dimension is a such one, that the dimension may be different at different points of the space, or the dimension may be not exist at all. It means that the definition of the topology and of the dimension independently of the world function is inconsistent, in general. Thus, the conventional method of the generalized geometry construction is overdetermined. It contains too many axioms, which are not independent. The suggested alternative method is insensitive to the continuity or discreteness of the geometry, as far as it uses nowhere the limiting transition or continuous coordinate system. A use of the conventional method of the geometry construction, when one postulates some axiom system, which determines the generalized geometry, appears to be ineffective, because it is very difficult to provide compatibility of original axioms. The demand of their compatibility imposes unnecessary constraints upon the obtained generalized geometries.

For instance, a construction of the Riemannian geometry may be realized by two methods. Using the conventional method on the basis of the metric tensor, giving on the n-dimensional manifold, one obtains the Riemannian geometry $\mathcal{G}_{\mathrm{R}}$. Using the alternative method based on the deformation principle, one obtains σ-Riemannian geometry $\mathcal{G}_{\sigma\mathrm{R}}$. Prefix σ means, that the generalized geometry $\mathcal{G}_{\sigma\mathrm{R}}$ has the property of σ-immanence (σ-immanence is the property of the geometry to be described completely by the world function σ). If the world function σ is the same in geometries $\mathcal{G}_{\mathrm{R}}$ and $\mathcal{G}_{\sigma\mathrm{R}}$, the obtained generalized geometries $\mathcal{G}_{\mathrm{R}}$ and $\mathcal{G}_{\sigma\mathrm{R}}$ are very close, but they distinguish in some details. For instance, in the σ-Riemannian geometry $\mathcal{G}_{\sigma\mathrm{R}}$ there is the absolute parallelism (fernparallelism), whereas in the Riemannian geometry $\mathcal{G}_{\mathrm{R}}$ one fails to introduce the absolute parallelism. The fact is that the concept of parallelism of two vectors in the Riemannian geometry $\mathcal{G}_{\mathrm{R}}$ is transitive according to the geometry construction [7, 2]. In the σ-Riemannian geometry $\mathcal{G}_{\sigma\mathrm{R}}$ there is an absolute parallelism, which is intransitive, but the parallelism of two vectors at the same point is transitive. Thus, if one introduces the absolute parallelism in the

Riemannian geometry, it will be intransitive, in general, and incompatible with the original statement on the transitivity of the parallelism in Riemannian geometry. To avoid inconsistency, one declares that in the framework of the Riemannian geometry one cannot introduce the absolute parallelism of vectors, placed at different points of the space.

The considered example shows that the conventional method of the generalized geometry construction is overdetermined. Furthermore, if the proper Euclidean geometry is considered to be as a special case of the Riemannian geometry, and one constructs it in the same way, as one constructs the Riemannian geometry, the proper Euclidean geometry acquires an absurd property. If some region of the space is nonconvex, the Riemannian geometry, constructed in this region with the Euclidean metric tensor, is not the Euclidean geometry, in general, because some distances in such a geometry are determined not along the straight lines of the Euclidean geometry, but along lines belonging partly to the boundary of the region. The obtained Riemannian space cannot be embedded, in general, isometrically into the Euclidean space, although the nonconvex region is a part of the Euclidean space. This absurd result shows, that the system of axioms of the conventional method of the generalized geometry construction is overdetermined. An application of the overdetermined method of the generalized geometry construction leads, in general, to inconsistency. The form of these inconsistencies depends essentially on the method, how these axioms are used.

3 On inconsistency of the Riemannian geometry

Some defects of the Riemannian geometry, were known years ago, but somehow they are not perceived as inconsistency of the axiom system, which determines the conventional construction of the Riemannian geometry. (Apparently, it is conditioned with the absence of the alternative method of the generalized geometry construction). For instance, if one constructs the Riemannian geometry as a special case of the Riemannian one, using the conventional method of the Riemannian geometry, the convexity problem appears, consisting in the fact, that a nonconvex region of the Euclidean plane cannot be embedded isometrically, in general, in the Euclidean plane, from which it is cut. The result is evidently absurd. However, for the convex region such an embedding is possible, mathematicians bypass this defect, considering geometries only on convex manifolds. (For instance, the book of A.D.Alexandrov "Internal geometry of convex surfaces"). Another defect is the problem of the fernparallelism, i.e. absence of definition of the parallelism of remote vectors in the Riemannian geometry. This fact is not considered as a defect of the Riemannian geometry also. In reality these defects are corollaries of the overdetermination of the Riemannian geometry, i.e. at the Riemannian geometry construction one uses more axioms, than it is necessary for the geometry construction. Some axioms are incompatible between themselves, or they are compatible only at some constraints, imposed on the geometry. In principle, this overdetermination may lead to another

contradictions, which are not known now. The overdetermination of the conventional method of the Riemannian geometry construction has been discovered after appearance of the alternative method, using essentially less amount of information, which is necessary for the geometry construction. The alternative method of the geometry construction does not contain overdetermination, the convexity problem, the problem of fernparallelism and other defects, which are corollaries of this overdetermination.

The main preconception, preventing the contemporary geometry from its development, is the statement that the straight is a one-dimensional line in any generalized geometry (the straight has no width) has been traced to Euclid. It is true in the Riemannian geometry for the straight, passing through the point P, in parallel to a vector at the point P. However, in the case, when the straight (geodesic) passes through the point P in parallel to a vector at another point P_1, it is not one-dimensional, in general, [5]. To avoid the non-one-dimensionality of the straight, the fernparallelism (i.e. the concept of parallelism of two remote vectors) is forbidden in the Riemannian geometry. The one-dimensionality of the straight may not be included in axiomatics of a generalized geometry. The one-dimensionality of the straight may not be used at the generalized geometry construction, because it may appear to be incompatible with other axioms. (Forbidding the fernparallelism, one did not think on a possible non-one-dimensionality of the straight. Simply an ambiguity in the definition of the parallelism contradicted to the axiomatics and seems to be unacceptable). Forbidding fernparallelism and the geometry on non-convex manifolds, one can avoid a manifestation of the Riemannian geometry inconsistency. However, it does not avoid the inconsistency in itself, because it may appear in other form. In the case of arbitrary generalized geometry the character of the straight (one-dimensional line, or multidimensional surface) is determined by the form of the world function, and one does need to make suppositions on the one-dimensionality, or on non-one-dimensionality of the straight. Furthermore, one cannot demand the one-dimensionality of the straight, because it leads to an overdetermination of conditions of the generalized geometry construction, and as a corollary to their inconsistency, or to a restriction of the class of possible geometries. Application of the topology at the generalized geometry construction supposes that a curve (and its kind a straight) is one-dimensional. It is a reason, why the topology may not be used at the generalized geometry construction. Overcoming of this preconception was very difficult. I personally needed almost thirty years for overcoming this preconception (a description of the path of this overcoming can be found in [1]), although I have an experience of successful overcoming of like preconception in other branches of physics.

Although the overcoming of the preconception on the one-dimensionality of the straight was difficult, I hoped, that, being elucidated it will be perceived and overcame by the mathematical community. It appeared, that I was wrong in this relation. The overcoming of the perception was very difficult (by the way, it was difficult also in the case of the preconception on the statistical description). Apparently, overcoming of preconceptions is always difficult. Besides, it appeared, that besides of

the main preconception on the one-dimensionality of the straight there are another preconceptions. For instance, one supposes, that any presentation of a geometry is a set of axioms and theorems and that the geometer's activity is a formulation and a proof of theorems.

4 Why mathematicians do not acknowledge T-geometry constructed on the basis of the deformation principle?

The relation of the most mathematicians to the alternative method of the geometry construction, which is based on the deformation principle is negative as a rule, although they cannot contradict anything against this method. Indeed, it as very difficult to contradict anything, because of the simplicity of the alternative method, which does not contain any suppositions except for the sufficiently evident deformation principle. Nevertheless, the refereed mathematical journals reject papers on the generalized geometry construction founded on basis of the deformation principle. In the paper [1] it is described in detail, how such a rejection is produced. My attempts of reporting these papers at the seminars of topologists - geometers are rejected also. However, my papers were reported and discussed at the seminar on the geometry as a whole in the Moscow Lomonosov University. This seminar was founded by N.V. Efimov.

In the present time the topological approach to the problem of the generalized geometry construction dominates. One supposes, that it is necessary at first to construct a proper topological space. In the topological space one introduces a metric and constructs a generalized geometry. Construction of the generalized geometries on the basis of the deformation principle depreciates essentially papers on geometry, and the mathematicians, constructing generalized geometries by the conventional method, were not enthused over the prospect. As far as they cannot suggest any essential objection against the deformation principle because of its simplicity and effectiveness, they use another methods of resistance, generating doubts in the scientific scrupulosity of advocates of the conventional method. One of them is described in [1] in detail.

Of course, I have known, how strongly we believe, that the straight line is always an one-dimensional geometrical object, because this preconception retarded my discovery of the T-geometry almost with thirty years. However, discovery of the non-one-dimensionality of the straight and incomprehension of such a possibility, when the non-one-dimensionality has been already discovered, seemed to be different things for me. It was appeared that a perception of the deformation principle is the more difficult, the better the person knows the geometry in its contemporary formal presentation. The fact is that the mathematicians perceive all things via the formalism. The associative perception like a reference to the deformation principle conveys little to them.

The strangers and the mathematicians perceive the geometry presentation as a formulation and proof of different geometric theorems. Replacement of theorems with definitions seems to them as anything quite obscure. However, in reality, the formulation and the proof of theorems is only one of the methods of work with different geometric statements (axioms and theorems). The proof of theorems is the most laborious part of work. As a result one gets an impression, that the geometry presentation is a proof of theorems (if there are no theorems, there is no geometry). But there are another methods of work with the geometric statements, which does not distinguish between the axioms and theorems.(There is no necessity to repeat the work of Euclid, one should use results of his work.) At such a method of work the theorems are replaced by definitions, and a necessity of their proof falls off.

I shall explain this in the example of the cosine theorem, which states

$$\begin{aligned} |\mathbf{BC}|^2 &= |\mathbf{AB}|^2 + |\mathbf{AC}|^2 - 2\left(\mathbf{AB}.\mathbf{AC}\right) \\ &= |\mathbf{AB}|^2 + |\mathbf{AC}|^2 - 2\,|\mathbf{AB}|\,|\mathbf{AC}|\cos\alpha \end{aligned} \tag{4.1}$$

where the points A, B, C are vertices of a triangle, $|\mathbf{BC}|$, $|\mathbf{AB}|$, $|\mathbf{AC}|$ are lengths of the triangle sides and α is the angle $\measuredangle BAC$. The relation (4.1) is the cosine theorem which is proved on the basis of the axioms of the proper Euclidean geometry.

Using expression of the length of the triangle side $\mathbf{AB}$ via the world function σ

$$|\mathbf{AB}| = \sqrt{2\sigma\left(A,B\right)} \tag{4.2}$$

we may rewrite the relation (4.1) in the form

$$\left(\mathbf{AB}.\mathbf{AC}\right) = \sigma\left(A,B\right) + \sigma\left(A,C\right) - \sigma\left(B,C\right) \tag{4.3}$$

The relation (4.3) is a definition of the scalar product $(\mathbf{AB}.\mathbf{AC})$ of two vectors $\mathbf{AB}$ and $\mathbf{AC}$ in the T-geometry. Thus, the theorem is replaced by the definition of a new concept (the scalar product), which is not connected directly with the concept of the linear space.

Another example the Pythagorean theorem for the rectangular triangle ABC with the right angle $\measuredangle BAC$, which is written in the form

$$|\mathbf{BC}|^2 = |\mathbf{AB}|^2 + |\mathbf{AC}|^2 \tag{4.4}$$

In T-geometry instead of the theorem (4.4) we have a definition of the right angle$\measuredangle BAC$. In terms of the world function this definition has the form. The angle $\measuredangle BAC$ is right, if the relation

$$\sigma\left(A,B\right) + \sigma\left(A,C\right) - \sigma\left(B,C\right) = 0 \tag{4.5}$$

takes place

Thus, we see that theorems of the proper Euclidean geometry are replaced by definitions of T-geometry.

From the formal viewpoint the difference between the conventional method of description and the alternative method may be described as follows. The conventional

method of the proper Euclidean geometry may be described as a set $\mathcal{S}_{\mathrm{E}}\left(\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\right]\right)$ of algorithms $\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\right]$, acting on operands $\mathcal{R}_{\mathrm{E}}$, where operands $\mathcal{R}_{\mathrm{E}}$ are geometrical objects or relations of the Euclidean geometry. The operands $\mathcal{R}_{\mathrm{E}}$ depend on parameters $\mathcal{P}^n \equiv \{P_0, P_0, .., P_n\}$, where $P_0, P_0, .., P_n$ are points of the space.

$$\mathcal{R}_{\mathrm{E}} = \mathcal{R}_{\mathrm{E}}\left(\mathcal{P}^n\right) \tag{4.6}$$

As far as all geometrical objects and relations $\mathcal{R}_{\mathrm{E}}$ can be expressed via the world function σ_{E} of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, one may rewrite the relation (4.6) in the form

$$\mathcal{R}_{\mathrm{E}} = \mathcal{R}_{\mathrm{E}}\left(\mathcal{P}^n\right) = \tilde{\mathcal{R}}_{\mathrm{E}}\left[\sigma_{\mathrm{E}}\left(\mathcal{P}^n\right)\right] \tag{4.7}$$

where $\sigma_{\mathrm{E}}\left(\mathcal{P}^n\right)$ is a set of world functions $\sigma_{\mathrm{E}}\left(P_i, P_k\right),\ P_i, P_k \in \mathcal{P}^n$.

Taking into account the relation (4.7), one can represent the set $\mathcal{S}_{\mathrm{E}}\left(\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\right]\right)$ of all algorithms $\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\left(\mathcal{P}^n\right)\right]$ in the form of the set $\mathcal{S}_{\mathrm{E}}\left(\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\left(\mathcal{P}^n\right)\right]\right) = \mathcal{S}_{\mathrm{E}}\left(\mathcal{A}\left[\tilde{\mathcal{R}}_{\mathrm{E}}\left[\sigma_{\mathrm{E}}\left(\mathcal{P}^n\right)\right]\right]\right) = \mathcal{S}_{\mathrm{E}}\left(\mathcal{A}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma_{\mathrm{E}}\right]\right]\right)$ of all algorithms $\widetilde{\mathcal{A}}\left[\sigma_{\mathrm{E}}\right] = \mathcal{A}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma_{\mathrm{E}}\right]\right]$. In the Euclidean geometry the set of all algorithms $\mathcal{S}_{\mathrm{E}}\left(\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\left(\mathcal{P}^n\right)\right]\right)$ takes the form $\mathcal{S}_{\mathrm{E}}\left(\widetilde{\mathcal{A}}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma_{\mathrm{E}}\right]\right]\right)$. In the generalized geometry $\mathcal{G}$, described by the world function σ, the set of all algorithms takes the form $\mathcal{S}_{\mathrm{E}}\left(\widetilde{\mathcal{A}}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma\right]\right]\right)$. It means that the set of all algorithms $\mathcal{S}_{\mathrm{E}}\left(\widetilde{\mathcal{A}}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma\right]\right]\right)$ for any generalized geometry is obtained from the set $\mathcal{S}_{\mathrm{E}}\left(\widetilde{\mathcal{A}}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma_{\mathrm{E}}\right]\right]\right)$ of all algorithms for the Euclidean geometry by means of replacement of the operand of algorithms $\sigma_{\mathrm{E}} \to \sigma$. The form of all algorithms is not changed. This is the formal description of the action of the deformation principle. For construction of the generalized geometry one needs no theorems and no logical reasonings. The main part of the work of construction of the generalized geometry consists in transformation of known Euclidean algorithms $\mathcal{A}\left[\mathcal{R}_{\mathrm{E}}\left(\mathcal{P}^n\right)\right]$ to the form $\widetilde{\mathcal{A}}\left[\tilde{\mathcal{R}}'_{\mathrm{E},\mathcal{P}^n}\left[\sigma_{\mathrm{E}}\right]\right]$, where all algorithms are represented in terms of the Euclidean world function.

In the T-geometry a construction of new generalized geometry is produced by the same algorithms, as in the proper Euclidean geometry. One replaces only the world function, i.e. operand of algorithms $\sigma_{\mathrm{E}} \to \sigma$. One does not need theorems which connect different objects of geometry. One uses the definition of the geometrical object or the relation of the Euclidean geometry, expressed directly via world function. For instance, the scalar product of two vectors is defined by the relation (4.3) in all T-geometries. The right angle $\measuredangle BAC$ is defined by the relation (4.5) in all T-geometries. There are no necessity to introduce couplings (theorems) between different geometrical concepts, as far as they are expressed via world function.

In T-geometry the main problem is an obtaining of expressions for different geometrical concepts and objects via the world function. As far as these expressions are the same for all T-geometries, it is sufficient to obtain these expressions in the framework of the proper Euclidean geometry. Thus, instead of formulation of

different theorems for different generalized geometries, the geometer must express all concepts and objects of the proper Euclidean geometry via the world function.

The formalism of T-geometry distinguishes essentially from the conventional formalism of the generalized geometry construction by the object of consideration. In the conventional formalism the geometrical objects and primarily the straight are objects of consideration. The conventional construction of the generalized geometry is a repetition of the Euclid's construction, which is produced on the basis of other axioms. The true choice of axioms is the main problem of the conventional method of the geometry construction. In the alternative method, based on the deformation principle, the object of consideration is the world function (but not geometrical objects), i.e. a function of two points of the space. It is essentially simpler object of consideration. However, the rules of work (logic of investigation [8]) with the world function are very complicated. It is very complicated to guess these rules, because they are determined by the Euclidean geometry. Primarily one guesses the rules of work with the most simple object: the straight line. As a matter of fact, they were not guessed, they have been taken from the Euclidean geometry. But, to take them, the Euclidean geometry is to be presented in terms of the world function. It means that it was necessary to construct the formalism on the basis of the world function. Construction of the formalism, founded on the world function, was difficult. It needed about forty years. Thirty years of these forty years were lost for the overcoming the preconception on the one-dimensionality of straight. Different stages of this formalism construction are described in [1].

Apparently, the most difficult and uncustomary for geometers - professionals are the facts, that a new object of consideration (world function) appears and the conventional objects of consideration (geometrical objects) turn into the logic (algorithm) of investigation. This transition from one method of investigation to another is difficult for perception of the geometers - professionals. However, it is essentially easier for non- professionals, who do not know, or know skin-deep the conventional method of the geometry construction. They do not need to compare both methods, overcoming complex of concepts of the conventional method and the formalism, connected with this method.

Constructing geometry on the basis of the deformation principle, it seems that the formalism is absent, in general, as far as the theorems are absent. (The geometers became accustomed that the geometric formalism appears only in the form of theorem.) In reality the formalism appears in the implicit form as a reference to the Euclidean geometry with its formalism. The formalism of the Euclidian geometry is modified at the replacement of the world function of the Euclidean space (Algorithms retain, only operand of algorithms is changed). Besides, for a realization of such a modification the Euclidean geometry is to be presented in the terms of the world function. Mathematicians know slightly this presentation, based on the world function formalism. Despite the evidence of the deformation principle, I obtained it, when the generalized geometry (T-geometry), based on its application, has been already constructed. Furthermore, appearance of the world function formalism (description of the Riemannian geometry in terms of the world function) takes priority

of the T-geometry construction.

5 Application of T-geometry physical problems

Besides, the overdetermination, appearing at a use of the conventional (topological) method of the geometry construction, this methods is not enough general. The method of the geometry construction based on the deformation principle, admits one to construct the generalized geometry, using deformation transforming the one-dimensional line into a surface. Although the everyday experience does not provide an example of such deformations, such deformations exist. Such a deformation may be described as follows. Let us imagine a beam of straight thin elastic wires of the same length. Being collected in a beam they represent a segment of the straight line. Let us grasp the ends of wires by two hands and move hands one to another. At such a deformation the wires diverge, conserving their lengths, and form an inswept surface. In other words, although deformations turning a one-dimensional line into a surface are possible, the Riemannian geometry and the metrical one ignore them, preferring to remain in the framework of deformations, conserving one-dimensionality of the straight. A use of the Euclidean concept of the straight, as a second base concept of the geometry, and consideration of the one-dimensionality of the straight do not admit one to prove out the deformation principle. The way to the deformation principle lies through the construction of the mathematical formalism, based on the application of the world function.

Applying the described above deformation to the construction of the space-time geometry [5], one succeeded to construct such a geometry, where the motion of free particles is primordially stochastic, and intensity of the stochasticity depends on the particle mass. After proper choice of the parameters of the space-time deformation (it depends on the quantum constant) the statistical description of free stochastic particles appears to be equivalent to the quantum description. [9]. It is important, that the quantum principles are not used. In other words, the quantum properties are descried by means of the correctly chosen space-time geometry. It is impossible in the framework of the Riemannian geometry as well as in the framework of any generalized geometry, constructed by means of the conventional method of the generalized geometry construction.

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